ANL-HEP-PR-00-83
hep-th/0008162

On the Finiteness of $\mathcal{N}=8$ Quantum Supergravity

Gordon Chalmers

Argonne National Laboratory
High Energy Physics Division
9700 South Cass Avenue
Argonne, IL 60439-4815

Abstract

We describe the constraints imposed on quantum maximal supergravity theories in perturbation theory by the two duality frameworks: S-duality in superstring theory and the AdS/CFT holographic correspondence between IIB superstring theory and $\mathcal{N}=4$ super Yang-Mills theory.

E-mail address: chalmers@pcl9.hep.anl.gov

1 Introduction

Constructing a theory of quantum gravity has motivated the formulation of both supergravity theories [1, 2] and superstring theories. Developments in the non-perturbative duality structures in superstring theories, in the art of calculating supergravity amplitudes, and in the formulation of a string theory description of $\mathcal{N}=4$ super Yang-Mills theory have allowed a fresh look at supergravity theories in different dimensions. In particular, maximal supergravities (preserving $\mathcal{N}=32$ supersymmetries) constructed in [3, 4, 5, 6] are re-examined in this context. In the past, and in the absence of any calculation ever producing a divergence in the maximally supersymmetric gravitational theory, $\mathcal{N}=8$ supergravity was superseded by the emergence of superstring theory, the non-perturbative form of which is presumably better defined than the former theory due to singularities. A counterterm compatible with the full $\mathcal{N}=8$ supersymmetry was initially constructed in [7, 8, 9].

Gravity theories in general dimensions have a dimensionful coupling constant that precludes the notion of perturbative renormalizability in the sense of Wilson. Duality in field and string theory predicts structures, however, in the coupling constants beyond what natural renormalization criteria suggests. Furthermore, duality structures are potentially leading to exact solutions of higher-dimensional theories, notably the large N_c limit of $\mathcal{N}=4$ super Yang-Mills gauge theory via tree-level scattering in IIB superstring theory. Finite coupling structure, or the intermediate coupling regime, is a corner of coupling space that is not related to the free-field limit by an inverse transformation in an infinitely large coupling constant in general, and these conclusions are based on investigations of this region in the parameter space.

In this letter we summarize the developments and predictions for the finiteness properties of maximally extended supergravity theories that have arisen through S-duality compatible graviton scattering[1] and in the holographic mapping between IIB superstring theory on anti-de Sitter space-times and boundary conformal field theories.

2 S- and U-duality in perturbation theory

The scattering of gravitons in Einstein frame, in accord with U-duality of IIB superstring (and M-) theory and to infinite genus order, must be invariant under the non-perturbative transformation of the string coupling constant. The form and the implications for $\mathcal{N}=8$ supergravity have been explored in [10]. The ten-dimensional

[1]The quantum IIB superstring theory is conjectured to be self-dual under fractional linear transformations of the coupling constant $\tau=\chi+ie^{-\phi}$. Although the truncation to the massless modes clearly does not remain S-duality invariant at the quantum level as can be found by explicit calculations, in contrast to the field equations of the supergravity, there is a remnant of the superstring structure that persists in the perturbative sector.

form of the graviton scattering in Einstein frame has the form within the derivative expansion,

$$S_{4-pt}^{\rm IIB} = \frac{1}{\alpha'} \sum_{k=0}^{\infty} \int d^{10}x\sqrt{g}\ \alpha'^k\ f_k(\tau,\bar\tau)\Box^k R^4 \ , \tag{2.1}$$

together with a series of non-analytic terms which may be reconstructed in a manifestly S-duality invariant fashion through unitarity relations leading to unitary scattering; the tensor structure associated with the placement of the derivatives $\Box^k$ in (2.1) is implied and not relevant for the analysis in this work although certainly so for a determination of the complete four-point scattering. The functions $f_k(\tau,\bar\tau)$ generically have an expansion in accord with the dilaton in perturbative string theory (and quantum supergravity) of

$$f_k(\tau,\bar\tau) = a_0^{(k)} \tau_2^{\frac{3}{2}+\frac{k}{2}} + a_1^{(k)} \tau_2^{-\frac{1}{2}+\frac{k}{2}} + a_2^{(k)} \tau_2^{-\frac{5}{2}+\frac{k}{2}} + \ldots\ . \tag{2.2}$$

The tensor R^4 is eight-derivatives and is an integral over all of on-shell constrained IIB superspace; its appearance in IIB string theory is well-known [11] and its tensor structure is a consequence of maximal supersymmetry. On the fundamental domain of $U(1)\backslash SL(2,R)/SL(2,Z)$, which labels the vacua of uncompactified IIB superstring theory and is parameterized by the space of coupling constants,

$$\mathcal{F}_1 = \Big\{ \tau=\tau_1+i\tau_2 : \tau_1^2+\tau_2^2 \geq 1\ ,\ \vert\tau_1\vert \leq \frac{1}{2} \Big\}\ , \tag{2.3}$$

any function invariant under $SL(2,Z)$ may be decomposed on the product of the set of functions,

$$E_s^{(q,-q)}(\tau,\bar\tau) = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^s}{(m+n\tau)^{s-q}(m+n\bar\tau)^{s+q}}\ , \tag{2.4}$$

where $\sum_i q_i = 0$ in the factors $E_{s_i}^{(q_i,-q_i)}$ (as in [10]), with $s=\sum_j s_j$, and with $s\geq 3/2$ or $s=\frac{1}{2}+it$ together with cusp forms possessing the expansion on the fundamental domain,

$$f_{\rm cusp}(\tau,\bar\tau) = \sum_{n\neq 0} a_n \tau_2^{\frac{1}{2}} K_{n-1/2}(2\pi\vert n\vert\tau_2) e^{2\pi i n\tau_1}\ , \tag{2.5}$$

with undetermined (but bounded) coefficients a_n [12] (no explicit examples of cusp forms are known and string theory may produce examples in the scattering). Eisenstein functions have appeared originally in the modular construction of the R^4 term involving D-instanton contributions [13] and in the context involving (p,q) string-instantons in [14] (see also [15] in more general cases). For large values of τ_2 the modular functions $E_s(\tau,\bar\tau)$ functions have the expansion, following a Poisson resummation,

$$E_s(\tau,\bar\tau) = 2\zeta(2s)\tau_2^s + 2\sqrt{\pi}\zeta(2s-1)\frac{\Gamma(s-\frac{1}{2})}{\Gamma(s)}\tau_2^{1-s}$$

$$+\frac{2\sqrt{\tau_2}\pi^s}{\Gamma(s)}\sum_{n\geq 1, m\neq 0}|\frac{m}{n}|^{s-\frac{1}{2}}K_{s-\frac{1}{2}}(2\pi|mn|\tau_2)e^{2\pi i m n\tau_1} \ , \tag{2.6}$$

with K_a the standard modified Bessel function. According to the Roelcke-Selberg decomposition [12] on the fundamental domain (2.3) any normalizable $SL(2,Z)$ function (with integration measure $d^2\tau/\tau_2^2$) can be written as

$$f(\tau,\bar\tau)=\sum_{n\geq 0} f_{cusp}^{(n)}(\tau,\bar\tau)\langle f_{\rm cusp}^{(n)},f\rangle+\frac{1}{4\pi i}\int_{t=\frac{1}{2}} dt\ E_{\frac{1}{2}+it}\langle E_{\frac{1}{2}+it},f\rangle \ , \tag{2.7}$$

with the integration along the imaginary axis and $\langle ...\rangle$ denoting an integration with measure $d^2\tau/\tau_2^2$. The difference between a normalizable and non-normalizable function is contained in the covariantization of the non-normalizable terms proportional to τ_2^s with $s\geq 3/2$; these covariantizations are listed in (2.4).[2]

The functions in (2.5)[3] have an expansion in powers of exponentials of the coupling constant and model D-instanton corrections to the amplitudes in the ten-dimensional case; these functions in (2.5) do not contribute to the perturbative expansion of either the superstring or in the supergravity limit. The set of functions in (2.4) and (2.5) span a basis on which all smooth $SL(2,Z)$ invariant functions may be expanded upon (for normalizable functions this is examined in [12]). The latter two sets of functions are L^2 normalizable on the fundamental domain, and the remaining (products of functions in (2.4) with $\sum_i q_i=0$) parameterize the $SL(2,Z)$ completion of the divergent pieces of a general $SL(2,Z)$ invariant function. This set of functions spans the space of the coefficents of the derivative expansion of the $SL(2,Z)$ invariant graviton scattering amplitude. This completes our summary of the general form of S-duality compliant graviton scattering previous to including further constraints; the functions $E_{1/2+it}$ individually have the large τ_2 expansion of $\tau_2^{1/2}\cos(\tau_2)$; potentially their contribution in (2.7) is compatible with the perturbative structure of string scattering through the transform in (2.7) but the non-perturbative instantonic corrections are not generally.

Furthermore, the scattering has been computed from the string up to genus two in the string-inspired supergravity approximation and twelve derivatives. We now turn to implementing the modular construction to $\mathcal{N}=4$ super Yang-Mills theory via the AdS/CFT correspondence at all values of the coupling constant.

[2]Further modular invariant functions may be deduced by enforcing the values of functions to be equal on the images of the fundamental domain under the fractional linear transformations. These examples, however, are singular and are excluded under the demand that the dilaton dependence of the string scattering is smooth on the fundamental domain. I thank E. Martinec for a discussion on this point.

[3]An explicit example of a cusp form has not been constructed to date, and string scattering may lead to examples in the non-perturbative regime.

3 Graviton scattering and AdS/CFT

The holographic correspondence between IIB superstring theory and $\mathcal{N}=4$ super Yang-Mills theory [16, 17, 18] is specified in part by the couplings of the $SU(N_c)$ gauge theory ($\tau_{\rm YM}=\frac{\theta_{YM}}{2\pi}+i\frac{4\pi}{g^2_{YM}}, N_c$) and string theory ($\tau=\frac{\theta}{2\pi}+\frac{i}{g_s}, \alpha'$) identifications [16]

$$\frac{R^2_{AdS}}{\alpha'}=\lambda\ , \qquad 4\pi g_s=g^2_{\rm YM}\ , \qquad \theta_s=\theta_{YM} \tag{3.1}$$

together with propagating string theory [18] on the $AdS_5\times S^5$ background with radius R_{AdS} and a non-vanishing five-form field flux in the AdS_5 directions,

$$F_{\mu_1\ldots\mu_5}=\frac{1}{R_{AdS}}\epsilon_{\mu_1\ldots\mu_5} \qquad R_{\mu_1\ldots\mu_4}=\frac{1}{R^2_{AdS}}(g_{\mu_1\mu_3}g_{\mu_2\mu_4}-g_{\mu_1\mu_4}g_{\mu_2\mu_3})\ . \tag{3.2}$$

Within the *strong* form of the correspondence the equivalence holds at all values of the couplings in (3.1)[4]. The map between gauge correlations of operators $\mathcal{O}_i(x_i)$ on the boundary and holographic string theory in the bulk is described in [18] in which the sources for the composite operators are the boundary data of higher-dimensional on-shell string fields $\phi_{o,i}$ via $S_{\rm int}=\int d^d\vec{x}\ \phi_{o,i}\mathcal{O}_i(x_i)$. An explicit coupling description of the genus expansion of the gauge theory through covariantized holographic string scattering has been explored in [19].

There are three regions in the parameter space of couplings that we explore in the gauge theory and IIB string theory:

$$\begin{array}{llll} I. & \lambda=g^2_{\rm YM}N_c & \text{finite} & N_c\ \text{large} \\ II. & \lambda=g^2_{\rm YM}N_c & \text{large} & g^2_{\rm YM}=\lambda/N_c\ \text{small} \\ III. & \lambda_D=\dfrac{N_c^2}{\lambda} & \text{finite} & N_c\ \text{large} \end{array} \tag{3.3}$$

The first limit is the expansion in the planar limit of the gauge theory, according to which the graphical expansion has the form $N_c^{2(1-\tilde{g})}F(\lambda)$ at string genus $\tilde{g}$; infinite N_c is described by tree-level string scattering in the AdS/CFT correspondence. The instanton corrections have fractional dependence on N_c and do not contribute as $N_c\to\infty$. The second limit describes strong coupling at large N_c described in the gauged supergravity approximation of the IIB superstring on $AdS_5\times S^5$ at small curvature according to the AdS/CFT duality. The third limit in (3.3) is the expansion in the S-dual variables of the gauge theory (and string theory); this expansion in the gauge theory is formulated in terms of the S-dual degrees of freedom, i.e. the monopoles and dyons in $\mathcal{N}=4$ super Yang-Mills theory. The coupling structure

[4] The integral functions upon which a correlation function, not restricted by conformal invariance in the four-point, may by expanded upon Appell functions from holographic string theory [19]; these arise in OPE analysis of conformal field theories [20] (and references therein)

has been explored for these limits in [19]. We further describe this latter limit in the following.

S-duality, in either $\mathcal{N} = 4$ super Yang-Mills theory or IIB superstring theory, exchanges the couplings through the S-generator as

$$\lambda \to \lambda_D = \frac{N^2}{\lambda} \ , \qquad N_c \to N_c \tag{3.4}$$

and does not commute with the large N_c planar expansion. S-duality exchanges fundamental fields within $\mathcal{N} = 4$ super Yang-Mills theory to those of the monopole and dyon degrees of freedom [21] (in the spontaneously broken case the theory exhibits the BPS mass formula $m^2 = 2|\vec{n}_e \cdot \vec{a} + \vec{n}_m \cdot \vec{a}_d|^2$ with ${a_d}^i = {\tau^i}_j a^j$; the unbroken limit of the theory contains an infinite number of massless monopoles and dyons despite the presence of a local microscopic Lagrangian)[5]. The strongly coupled limit of the monopole and dyon formulation in the large N_c limit is the expansion of

$$\lambda/N_c^2 \sim \text{ small} \qquad N_c \sim \text{ large} \ . \tag{3.5}$$

We remind the reader that strongly coupled monopole and dyon formulation of $N = 4$ super Yang-Mills theory involve the limit in (3.5), i.e. $g^2 \to 0$, and this expansion describes the planar limit of the gauge theory in terms of these soliton degrees of freedom, leading to a different expansion in the coupling constants, i.e. λ/N_c^2 and N_c.

This dual expansion of the gauge theory is examined within the holographic anti-de Sitter correspondence in [19]. The genus truncation is related to the existence of this limit of the gauge theory, according to which the limit exists, and given the S-duality of the gauge theory implies that the maximum number of genus corrections to the order α'^k in the derivative expansion is $g_{\text{max}} = \frac{1}{2}(k+2)$ for k even and $g_{\text{min}} = \frac{1}{2}(k+1)$; example listings are in Table 1. The genus truncation directly prohibits terms in the scattering that diverge as N_c becomes large at finite dual coupling [19]. This form also agrees with the expansion of modular invariant functions in accord with IIB superstring theory with the inclusion of the multiplicative product of zero-weight functions $E_s^{(q,-q)}(\tau,\bar{\tau})$ for $s \geq 3/2$ and real, described in [10].

Apart from the two dualities, a proof of this property of the expansion of the S-matrix might be useful for providing information in the opposite direction, that is, to further a proof of S-duality. At the genus two expansion of the four-point graviton scattering amplitude [22], the integrand appears to have an R^4 eight-derivative term that is in contradiction to explicit two-loop supergravity calculations. This term in string theory must integrate to zero and is entirely proportional to the extra

[5]The dual description may be obtained by inverting the spontaneously broken theory in the coupling and then taking the vacuum values of the scalar fields to zero. In this manner the spectrum of massive states is well-defined in the local sense.

Table 1: Example contributions to $\Box^k R^4$ arising in string perturbation theory at genus g. The asterisk denotes the top $g_{\max}$ genus contributions.

	$g=0$	$g=1$	$g=2$	$g=3$	$g=4$	$\ldots$
R^4	$\surd$	$\surd^\star$				
$\Box R^4$						
$\Box^2 R^4$	$\surd$		$\surd^\star$			
$\Box^3 R^4$	$\surd$	$\surd$	$\surd$			
$\Box^4 R^4$	$\surd$	$\surd$	$\surd$	$\surd^\star$		
$\Box^5 R^4$	$\surd$	$\surd$	$\surd$	$\surd$		
$\Box^6 R^4$	$\surd$	$\surd$	$\surd$	$\surd$	$\surd^\star$	
$\ldots$						

superconformal ghost degrees of freedom [23] (i.e. picture structure) that must be inserted in the amplitude calculation at higher genera. This suggests, at $g=2$ and similar analysis at $g>2$ (one limit via factorizing multi-genus on products of $g=2$), that a justification of the duality structure of the scattering might be found in an appropriate interpretation in the target space-time of the world-sheet BRST invariance. These zero-momentum operators, in the field theory limit, could be associated with generating cancellations in the target space-time graviton scattering leading to the genus truncation in string perturbation theory.

The genus truncation implied by the two independent dualities of string theory in the low-energy scattering implies cancellations of the graviton scattering in the maximally extended setting. No divergence has ever been produced in the scattering of maximal supergravity and the four-point graviton function at two-loops has been determined [24]. This theory may be formulated as the toroidal compactification and reduction to massless modes of $\mathcal{N}=1$ $d=11$ supergravity or $\mathcal{N}=2$ $d=10$ IIB supergravity (the complete reduction of which can explicitly be shown to agree with perturbative string theory amplitude calculations up to genus two).

The U-duality group $E_{11-d(11-d)}$, which conjecturally connects the five fundamental string theories [25], contains an S-duality subgroup for values of d which inverts fractionally the dilaton coupling constant. We consider the special point of the moduli of the toroidally compactified theory when all moduli except for the ten-dimensional dilaton have vanishing values (this does not lead to enhanced gauge symmetry as the multiplet containing the graviton also contains the vector particles). In the toroidal limit the large τ_2 behavior of the Eisenstein series may be constructed. The functional form of the small moduli contribution is invariant under $SL(2,Z)$ transformations and is for large values of τ_2,

$$E_s^{E_{11-d(11-d)}}(\tau,\bar\tau) = \left[\tau_2 V_{10-d}^{\frac{4}{10-d}}\right]^{\left(\frac{s(10-d)}{(d-2)}\right)} \sum_{(p,q)\neq(0,0)} \frac{\tau_2^s}{|p+q\tau|^{2s}} + \ldots \ , \tag{3.6}$$

the form of which truncates in every dimension, in the same manner as it does in $d = 10$ (the volume factor is inert under duality transformations and represents the dimensionality of the d-dimensional gravitational coupling constant). The form in (3.6) is due to the $SL(2,Z)$ subgroup of the larger U-duality group. Constructions based upon the generalized Eisenstein series follows similarly in the toroidal compactifications (and in fractional dimensions d), with the difference involving the volume factor of the $10-d$ dimensional tori. The perturbative truncation persists in all of the toroidal compactifications and in every dimension.

4 String theory to gravity field theory

The agreement of the low-energy field theory modelling of the superstring requires an infinite number of cancellations or nullifications at higher genus for every $\Box^k$ term in the low-energy expansion. The genus zero and two contributions to the $\Box^2 R^4$ term in the ten-dimensional S-matrix, for example, indicates that separately every term proportional to $\Box^2$ at $g > 2$ has to be zero (every perturbative term at different genus differs by powers of τ_2^2). The same structure occurs because of the truncation $g^k_{\rm max} = \frac{1}{2}(k+2)$ and $g^k_{\rm min} = \frac{1}{2}(k+1)$, for even and odd k, for the higher derivative terms in the expansion of the scattering (and in toroidal compactifications preserving the maximal supersymmety). The two-loop integral form of the four-graviton scattering has been computed explicitly within field theory in [24], and via unitarity constructions to be finite up to five loops in $d = 4$ [24]. We describe the continuation of the predictions of duality at an infinite number of loops in this letter, and we compare with the known explicit calculations.

We analyze in the following the first few terms in the S-matrix expansion to outline the cancellations; we discuss $d = 10$ with a similar analysis in compactified dimensions. In the IIB supergravity limit of the superstring scattering, primitive divergences in $d = 10$ of the four-point amplitude at one-loop are of the form, with coefficients determined by the domain of integration inherited from the string moduli space,

$$A_4^{L=1,m=0} \sim \Lambda^2 R^4 + \Box R^4 + \ldots \tag{4.1}$$

$$A_4^{L=1,m\neq 0} \sim \Box R^4 + \alpha' \Box^2 R^4 + \ldots \ , \tag{4.2}$$

from an explicit supergravity one-loop calculation or arising from the low-energy expansion of the genus one amplitude [11]. The α' in (4.2) indicates an overall factor of two derivatives, a $\Box$ or s_{ij} in momentum space, in comparison to the massless modes after they are normalized correctly at the given order.

At two-loops in the supergravity theory, an explicit twelve derivatives may be extracted from the loop integration [24] and the amplitude has the generic tensor

structure,

$$A_4^{L=2,m=0} \sim \Box^2 \left(\Lambda^6 + \Lambda^4 \Box + \dots\right) R^4 \ . \tag{4.3}$$

Explicit string theory calculations at genus two may be computed in the $\alpha' \to 0$ limit [22]. The massive modes of the string contribute at an order α' higher in the low-energy limit (as at order $g=0$ and $g=1$), and an additional pair of derivatives must also be extracted,

$$A_4^{L=2,m\neq 0} \sim \Box^3 R^4 + \dots \ . \tag{4.4}$$

This is in agreement with the conjecture for the $\Box^2 R^4$ term in eleven dimensions [26] upon dimensional reduction and T-dualized to IIB superstring theory in ten dimensions.

At tree and one-loop level the massive modes explicitly contribute an order higher in α', and such a property persists at higher order in the string-inspired regulator. This may also be analyzed via decoupling and unitarity considerations (superconformal ghost insertions complicate an analysis directly in the string scattering). Via decoupling, the domain of integration in the generalized Schwinger proper-time regulator captures both the UV behavior of both the massless and massive modes. For example, at genus one the integration is over the fundamental domain in (2.3). The string theory is not directly an infinite summation over the massive fields, but upon comparing the integration of tower of string states, the field theory in the string-inspired regulator models the UV behavior. The string-inspired regulator applied to the massless modes reflects the massive modes through the definition associated with the integration over the moduli of the world-sheet (the integration region associated with integrating over the world-line of a propagating field).

Multi-loop graviton scattering generically produces two-derivatives at every vertex that must be integrated in individual diagrams. The perturbative counting arising from the Eisenstein series construction, i.e. $g_{\rm min}=0$ to $g^k_{\rm max}$, has the same behavior in the field theory by an extraction of an additional four derivatives at every successive loop order (as found for example at loop order one and two). We denote the number of cancelled components of the internal tensor associated with derivative vertices by $N_L = 4(1+L)$; such a structure in Feynman diagrams also arises in ϕ^3 theory. In the limit in which the ladder diagrams are isolated, and in the conjectured construction of [24], this property is seen to infinite loop order (before integration) for the massless modes, and pure $\mathcal{N}=8$ supergravity at vanishing moduli is perturbatively finite in this regime to all orders in four through six dimensions. The dualities indicate the same structure after summing over *all* perturbative diagrams at every loop order; complete three-loop expressions based on quantizing the $N=8$ theory for graviton scattering have not been obtained yet although many contributions have been isolated through cut-constructibility in [24]. Previous work

based on the hypothetical existence of an unconstrained off-shell superspace indicates cancellations up to seven loops [27] (although such a formalism containing only a finite number of fields has not been constructed). These cancellations via the Eisenstein formulae occur in every integral dimension; however, the domain of integration over the supergravity modes in first quantized form over the world-line parameters is unchanged dimensionally.

The tensor property, i.e. an extraction or nullification of internal loop momenta associated with the gravitational couplings in the pattern $4(1 + L)$, implies that $\mathcal{N} = 8$ supergravity with all moduli tuned to zero except for the dilaton coupling is finite in four through six dimensions. The extension of S-duality to general values of dimension generates regulator independence in the predictions. A calculation of the primitive divergence in maximal supergravity at three-loops in ten dimensions is useful to further establish the modular invariance predictions of S-duality in the superstring truncated to the massless modes. The graviton scattering indicates that this primitive divergence is suppressed by four powers and is of order Λ^{10}, the first cancellation in a series related to to the counting of reduced tensor structure $4(1+L)$ at L loops.

Given the two dualities, both S-duality (and U-duality) and the AdS/CFT duality, at finite values of couplings the supersymmetric perturbative quantum theory of gravity in the maximally supersymmetric case appears finite. These dualities have not been proved; however, there is evidence for both of these dualities in the extension to arbitrary energies and couplings.

Acknowledgements

The work of G.C. is supported in part by the U.S. Department of Energy, Division of High Energy Physics, Contract W-31-109-ENG-38. G.C. thanks Carlos Wagner for encouragement in this work.

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